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# THE DAVIS–GUT LAW AND LAI LAW FOR FINITELY INHOMOGENEOUS WALKS

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*Abstract.* Let $\{X_n, \geqslant 1\}$ be a sequence of independent and identically distributed random variables with partial sums $S_n = \sum_{k=1}^n X_k$, $n \geqslant 1$. Davis-Gut law states that

$$\sum_{n=1}^{\infty} \frac{1}{n} P\left\{|S_n| > (1+\varepsilon)\sqrt{2n\log\log n}\right\} \begin{cases} < \infty, & \text{if } \varepsilon > 0, \\ = \infty, & \text{if } \varepsilon < 0 \end{cases}$$

if and only if $EX_1 = 0$ and $EX_1^2 = 1$. Lai law states that

$$\sum_{n=1}^{\infty} n^{r-1} P\{|S_n| > (1+\varepsilon)\sqrt{2rn\log n}\} \begin{cases} < \infty, & \text{if } \varepsilon > 0, \\ = \infty, & \text{if } \varepsilon < 0 \end{cases}$$

if and only if $EX_1 = 0$, $EX_1^2 = 1$ and $E(X_1^2/\log|X_1|)^{r+1} < \infty$, where $r > 0$. The paper will extend those results to the case where $\{X_n, n \geqslant 1\}$ are no longer identically distributed, but rather their distributions come from a finite set of distributions.

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