

IMPROVED JENSEN–TYPE INEQUALITIES VIA LINEAR INTERPOLATION AND APPLICATIONS

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Abstract. In this paper we develop a general method for improving Jensen-type inequalities for convex and, even more generally, for piecewise convex functions. Our main result relies on the linear interpolation of a convex function. As a consequence, we obtain improvements of some recently established Young-type inequalities. Finally, our method is also applied to matrix case. In such a way we obtain improvements of some important matrix inequalities known from the literature.

Mathematics subject classification (2010): 26D15, 26A51, 15A45.

Keywords and phrases: Convex function, Jensen inequality, Young inequality, Kantorovich constant, Specht ratio, arithmetic mean, geometric mean, Heinz mean.

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