

ON TWO BIVARIATE ELLIPTIC MEANS

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Abstract. This paper deals with the inequalities involving the Schwab-Borchardt mean SB and a new mean N introduced recently by this author. In particular optimal bounds, for SB are obtained. Inequalities involving quotients N/SB , for the data satisfying certain monotonicity conditions, are derived.

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