Journal of Mathematical Inequalities
Volume 11, Number 2 (2017), 469–483
doi:10.7153/jmi-11-39

THE STEFFENSEN–POPOVICIU MEASURES IN THE CONTEXT OF QUASICONVEX FUNCTIONS

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Abstract. We discuss the extension of Jensen's inequality to the framework of quasiconvex functions and of signed measures.

Mathematics subject classification (2010): 26A51, 26B25, 26D15, 28A10.
Keywords and phrases: Jensen's inequality, convex function, quasiconvex function, signed measure, Rayleigh measure, Cauchy's interlace theorem.

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