

## THE STEFFENSEN–POPOVICIU MEASURES IN THE CONTEXT OF QUASICONVEX FUNCTIONS

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*Abstract.* We discuss the extension of Jensen’s inequality to the framework of quasiconvex functions and of signed measures.

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