

## SOME NEW VOLTERRA–FREDHOLM TYPE DISCRETE INEQUALITIES WITH FOUR ITERATED INFINITE SUMS WITH APPLICATIONS

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*Abstract.* Some generalized Volterra-Fredholm type nonlinear discrete inequalities involving four iterated infinite sums are discussed. Some new explicit bounds for solutions of certain difference equations are derived. To illustrate the significance of results, some new applications are given.

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