

NEW INEQUALITIES FOR THE VOLUME OF THE UNIT BALL IN \mathbb{R}^n

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Abstract. Many interesting monotonicity properties and inequalities for the volume of the unit ball in \mathbb{R}^n have been established. The main object of this paper is to establish new inequalities for the volume of the unit ball in \mathbb{R}^n .

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