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MAPPING PROPERTIES OF HARDY–TYPE OPERATORS INVOLVING GENERAL FUNCTIONS

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Abstract. Weight characterizations are obtained for the boundedness and compactness of the operator

$$(Sf)(x) = u_1(x)\int_{a(x)}^{b(x)} v_1(t)f(t)dt + u_2(x)\int_{c(x)}^{d(x)} v_2(t)f(t)dt,$$

where $u_i, v_i, i = 1,2$, are certain general measurable functions (not necessarily non negative), between weighted Lebesgue spaces $L^p(I,w_0)$ and $L^q(I,w_1)$, where $1 < p,\ q < \infty$ and $I = (0,\infty)$.

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