

## MAPPING PROPERTIES OF HARDY-TYPE OPERATORS INVOLVING GENERAL FUNCTIONS

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*Abstract.* Weight characterizations are obtained for the boundedness and compactness of the operator

$$(Sf)(x) = u_1(x) \int_{a(x)}^{b(x)} v_1(t)f(t)dt + u_2(x) \int_{c(x)}^{d(x)} v_2(t)f(t)dt,$$

where  $u_i, v_i, i = 1, 2$ , are certain general measurable functions (not necessarily non negative), between weighted Lebesgue spaces  $L^p(I, w_0)$  and  $L^q(I, w_1)$ , where  $1 < p, q < \infty$  and  $I = (0, \infty)$ .

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