

EXISTENCE AND UNIQUENESS SOLUTIONS FOR A CLASS OF HEMIVARIATIONAL INEQUALITIES

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Abstract. This paper deals with the existence and uniqueness of results for a class of hemivariational inequality problem.

$$\beta_1(x, y) + \beta_2(x, y) + J^0(x; y - x) \geq 0.$$

Moreover, we enhance the main results an application to the existence of solution for a differential inclusion.

Mathematics subject classification (2010): 47H04, 47H05, 47J20, 49J53, 26D15.

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REFERENCES

- [1] B. ALLECHE, V. RĂDULESCU, M. SEBAOUI, *The Tikhonov regularization for equilibrium problems and applications to quasi-hemivariational inequalities*, Optim., **9** (2015) 483–503.
- [2] I. ANDREI AND N. COSTEA, *Nonlinear hemivariational inequalities and applications to nonsmooth mechanics*, Adv. Nonlinear Var. Inequal. **13** (2010) 1–17.
- [3] J. P. AUBIN, AND F. H. CLARKE, *Shadow prices and duality for a class of optimal control problems*, SIAM J. Control Optim. **17** (1979) 567–586.
- [4] M. BERGER, *Nonlinearity and Functional Analysis*, Academic Press, New York (1977).
- [5] E. BLUM AND W. OETTLI, *From optimization and variational inequalities to equilibrium problems*, The Mathematics Student **63** (1994) 123–145.
- [6] H. BREZIS, *Analyse Fonctionnelle: Théorie et Applications*, Masson, Paris (1992).
- [7] F. E. BROWDER, *The solvability of non-linear functional equations*, Duke Math. J. **30** (1963) 557–566.
- [8] F. H. CLARKE, *Optimization and Nonsmooth Analysis*, Wiley (1983).
- [9] N. COSTEA AND V. RĂDULESCU, *Existence results for hemivariational inequalities involving relaxed $\eta - \alpha$ monotone mappings*, Commun. Appl. Anal. **13** (2009) 293–304.
- [10] K. FAN, *A generalization of Tychonoffs fixed point theorem*, Math. Ann. **142** (1961) 305–310.
- [11] Y. P. FANG AND N. J. HUANG, *Variational-like inequalities with generalized monotone mappings in Banach spaces*, J. Optim. Theory Appl. **118** (2003) 327–338.
- [12] P. HARTMAN AND G. STAMPACCHIA, *On some nonlinear elliptic differential functional equations*, Acta Math. **115** (1966) 271–310.
- [13] A. E. HASHOOSH AND M. ALIMOHAMMADY, *On Well-Posed of Generalized Equilibrium Problems Involving α -Monotone Bifunction*, Journal of Hyperstructures **5** (2016), 151–168.
- [14] A. E. HASHOOSH, M. ALIMOHAMMADY AND M. K. KALLEJI, *Existence Results for Some Equilibrium Problems involving α -Monotone Bifunction*, International Journal of Mathematics and Mathematical Sciences, **2016** (2016) 1–5.
- [15] U. KAMRAKSA AND R. WANGKEEEREE, *Generalized equilibrium problems and fixed point problems for nonexpansive semigroups in Hilbert spaces*, Journal of Global Optimization, **51** (2011) 689–714.
- [16] B. KNASTER, K. KURATOWSKI AND S. MAZURKIEWICZ, *Ein Beweis des Fixpunktsatzes für n-dimensionale Simplexe*, Fund. Math. **14** (1929) 132–137.

- [17] N. K. MAHATO AND C. NAHAK, *Mixed equilibrium problems with relaxed α -monotone mapping in Banach spaces*, Rendiconti del Circolo Matematico di Palermo, (2013).
- [18] D. MOTREANU, P. D. PANAGIOTOPOULOS, *Minimax theorems and qualitative properties of the solutions of hemivariational inequalities*, Nonconvex Optimization and its Applications **29**, Kluwer Academic Publishers, Dordrecht, 1999.
- [19] D. MOTREANU, V. RĂDULESCU, *Variational and non-variational methods in nonlinear analysis and boundary value problems*, Nonconvex Optimization and its Applications **67**, Kluwer Academic Publishers, Dordrecht, 2003.
- [20] Z. NANIEWICZ, P. D. PANAGIOTOPOULOS, *Mathematical theory of hemivariational inequalities and applications*, Monographs and Textbooks in Pure and Applied Mathematics **188**, Marcel Dekker, Inc., New York, 1995.
- [21] P. D. PANAGIOTOPOULOS, *Nonconvex energy functions. Hemivariational inequalities and substationarity principles*, Acta Mech. **42** (1983) 160–183.
- [22] P. D. PANAGIOTOPOULOS, M. R. FUNDO AND V. RĂDULESCU, *Existence theorems of Hartman-Stampacchia type for hemivariational inequalities and applications*, J. Global Optim. **15** (1999) 41–54.
- [23] J. W. PENG AND J. YAO, *A viscosity approximation scheme for system of equilibrium problems, nonexpansive mappings and monotone mappings*, Nonlinear Anal. **71** (2009) 6001–6010.
- [24] V. RĂDULESCU, D. REPOVŠ, *Existence results for variational-hemivariational problems with lack of convexity*, Nonlinear Anal. **73** (2010) 99–104.
- [25] V. RĂDULESCU, D. REPOVŠ, *Partial Differential Equations with Variable Exponents: Variational Methods and Qualitative Analysis*, CRC Press, Taylor Francis Group, Boca Raton FL (2015).
- [26] D. REPOVŠ AND C. VARGA, *A Nash type solutions for hemivariational inequality systems*, Nonlinear Analysis **74** (2011), 5585–5590.
- [27] A. TADA AND W. TAKAHASHI, *Weak and strong convergence theorems for a nonexpansive mapping and equilibrium problem*, J. Optim. Theory Appl. **133** (2007) 359–370.
- [28] E. TARAFDAR, *A fixed point theorem equivalent to the Fan-Knaster-Kuratowski-Mazurkiewicz Theorem*, J. Math. Anal. Appl. **2** (1987) 475–479.
- [29] R. U. VERMA, *A-monotonicity and its role in nonlinear variational inclusions*, J. Optim. Theory Appl. **129** (2006) 457–467.
- [30] R. U. VERMA, *On generalized variational inequalities involving relaxed Lipschitz and relaxed monotone operators*, Journal of Mathematical Analysis and Applications, **213** (1997) 387–392.
- [31] R. U. VERMA, *On monotone nonlinear variational inequality problems*, Commentationes Mathematicae Universitatis Carolinae, **39** (1998) 91–98.