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ENDPOINT ESTIMATES FOR COMMUTATORS OF SUBLINEAR OPERATORS IN THE MORREY–TYPE SPACES

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Abstract. Let $[b,\mathscr{T}_\alpha]$ $(0\leqslant\alpha<n)$ be the commutators generated by $BMO(\mathbb{R}^n)$ functions and a class of sublinear operators satisfying certain size conditions. The aim of this paper is to study the endpoint estimates of these commutators on the weighted Morrey spaces and the generalized Morrey spaces, under the assumptions that $[b,\mathscr{T}_\alpha]$ $(0\leqslant\alpha<n)$ satisfy (weighted or unweighted) endpoint inequalities on $\mathbb{R}^n$ or on bounded domains. Furthermore, as applications of our main results, we will obtain, in the endpoint case, the boundedness properties of many important operators in classical harmonic analysis on the weighted Morrey and the generalized Morrey spaces.

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