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ON A NEW FAMILY OF BIVARIATE MEANS

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Abstract. A new family of bivariate means is defined and investigated. Members of that family are generated by the Schwab-Borchardt mean. Comparison results involving new means and the second Neuman mean are established. In particular, two means introduced and studied by J. Sándor and Z. Yang belong to a new class of means.

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