

ON SOME MULTILINEAR COMMUTATORS IN VARIABLE LEBESGUE SPACES

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Abstract. In this paper, the authors obtain some characterizations of BMO in terms of commutators of multilinear fractional integrals and Calderón-Zygmund singular integrals on variable Lebesgue spaces. The corresponding weighted estimates for vector-valued commutators and multilinear commutators with vector symbol on variable Lebesgue spaces are also considered.

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