

ON ROUGH GENERALIZED PARAMETRIC MARCINKIEWICZ INTEGRALS

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Abstract. We obtain certain sharp L^p bounds for the generalized parametric Marcinkiewicz integrals $\mathcal{M}_{\Omega, h, \rho}^{(\lambda)}$. The singular kernels are allowed to be rough on the unit sphere as well as in the radial direction. By the virtue of these estimates along with an extrapolation argument we obtain some new and improved results on generalized parametric Marcinkiewicz integrals. Our conditions on Ω and h are known to be the weakest conditions in their respective classes. One of our main results answers a question posed by Fan and Wu.

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