Journal of Mathematical Inequalities
Volume 11, Number 3 (2017), 817–829
doi:10.7153/jmi-2017-11-63

A METHOD FOR PROVING SOME INEQUALITIES ON MIXED HYPERBOLIC–TRIGONOMETRIC POLYNOMIAL FUNCTIONS

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Abstract. In this article we present a method for proving inequalities of the form

$$f(x)=\sum_{i=1}^{n}\alpha_i x^{p_i}\sinh^{q_i}x\cosh^{r_i}x>0,$$

for $x\in(\delta_1,\delta_2)$, $\delta_1\leqslant 0\leqslant\delta_2$; where $\alpha_i\in\mathbb{R}\setminus\{0\}$, $p_i,q_i,r_i\in\mathbb{N}_0$ and $n\in\mathbb{N}$. The method is based on the precise approximations of the hyperbolic sine and hyperbolic cosine functions by Maclaurin polynomials. Using this method we present new proofs of some well-known inequalities, but also we prove some new inequalities. Inequalities involving hyperbolic functions are much less studied than inequalities involving trigonometric functions. In this paper, the method described in the article [7] has been adapted to the inequalities involving hyperbolic functions.

Mathematics subject classification (2010): 26D05, 41A10.
Keywords and phrases: Hyperbolic inequalities, trigonometric inequalities, approximations of the hyperbolic sine and hyperbolic cosine.

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Paper JMI-11-63