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# ESTIMATIONS OF POWER DIFFERENCE MEAN BY HERON MEAN

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*Abstract.* As generalizations of arithmetic and geometric means, for positive real numbers $a$ and $b$, power difference means $J_q(a,b) = \frac{q}{q+1}\frac{a^{q+1}-b^{q+1}}{a^q-b^q}$ and Heron means $K_q(a,b) = (1-q)\sqrt{ab} + q\frac{a+b}{2}$ are well known. In this paper, we obtain the greatest value $\alpha = \alpha(q)$ and the least value $\beta = \beta(q)$ such that the double inequality

$$K_\alpha(a,b) < J_q(a,b) < K_\beta(a,b)$$

holds for any $q \in \mathbb{R}$, which includes Xia, Hou, Wang and Chu's result. Moreover, from this result, we derive operator inequalities for bounded linear operators on a Hilbert space.

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