

## VARIABLE EXPONENT SOBOLEV SPACES ASSOCIATED WITH JACOBI EXPANSIONS

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*Abstract.* In this paper we define variable exponent Sobolev spaces associated with Jacobi expansions. We prove that our generalized Sobolev spaces can be characterized as variable exponent potential spaces and as variable exponent Triebel-Lizorkin type spaces.

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