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ON THE RESTRICTED SUMMABILITY OF THE MULTI–DIMENSIONAL VILENKIN–CESÀRO MEANS

ISTVÁN BLAHOTA AND KÁROLY NAGY

Abstract. The properties of the maximal operator of the (C,α)-means ($\alpha=(\alpha_1,\dots,\alpha_d)$) of the multi-dimensional Vilenkin-Fourier series are discussed, where the set of indices is inside a cone-like set. Weisz proved that the maximal operator is bounded from martingale Hardy space H_p^γ to the space L_p for $p_0<p$ ($p_0=\max\{1/(1+\alpha_k);k=1,\dots,d\}$) [21]. The next question arise. Is the boundary point p_0 essential or not? In the present paper we show that the maximal operator $\sigma_L^{\alpha,*}$ is not bounded from the Hardy space $H_{p_0}^\gamma$ to the space L_{p_0}.

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REFERENCES

[1] G. N. AGAJEV, N. YA. VILENKIN, G. M. DZHAFARLI, A. I. RUBINSTEIN, *Multiplicative systems of functions and harmonic analysis on 0-dimensional groups*, "ELM" (Baku, USSR) (1981) (Russian).
[2] I. BLAHOTA, G. GÁT, *Pointwise convergence of double Vilenkin-Fejér means*, Studia Sci. Math. Hungar. **36** (2000), 49–63.
[3] N. J. FINE, *Cesàro summability of Walsh-Fourier series*, Proc. Nat. Acad. Sci. USA **41** (1955), 588–591.
[4] G. GÁT, *On the divergence of the* $(C,1)$ *means of double Walsh series*, Proc. Amer. Math. Soc. **128**, 6 (1996), 1711–1720.
[5] G. GÁT, *Pointwise convergence of cone-like restricted two-dimensional* $(C,1)$ *means of trigonometric Fourier series*, J. Approx. Theory **149** (2007), 74–102.
[6] G. GÁT, *Pointwise convergence of the Cesàro means of double Walsh series*, Annales Univ. Sci. Budapest., Sect. Comp. **16** (1996), 173–184.
[7] G. GÁT, K. NAGY, *Pointwise convergence of cone-like restricted two-dimensional Fejér means of Walsh-Fourier series*, Acta Math. Sinica, English Series **26**, 12 (2010), 2295–2304.
[8] U. GOGINAVA, *The maximal operator of the* (C,α) *means of the Walsh-Fourier series*, Annales Univ. Sci. Budapest., Sect. Comput. **26** (2006), 127–135.
[9] U. GOGINAVA, *Maximal* (C,α,β) *operators of two-dimensional Walsh-Fourier series*, Acta Math. Acad. Paed. Nyíregyház. **24** (2008), 209–214.
[10] F. MÓRICZ, F. SCHIPP, W. R. WADE, *Cesàro summability of double Walsh-Fourier series*, Trans. Amer. Math. Soc. **329**, 1 (1992), 131–140.
[11] K. NAGY, *On the restricted summability of double Walsh-Fourier series*, Publ. Math. (Debrecen) **85**, 1–2 (2014), 113–122.
[12] K. NAGY, *Almost everywhere convergence of cone-like restricted two-dimensional Fejér means with respect to Vilenkin-like systems*, St. Petersburg Mathematical Journal **25**, 4 (2014), 605–614. Originally published in Algebra i Analiz **25**, 4 (2013), 125–138.
[13] K. NAGY, *Almost everywhere convergence of cone-like restricted double Fejér means on compact totally disconnected groups*, Acta Math. Acad. Paed. Nyíregyház. **27** (2011), 267–277.
[14] L.-E. PERSSON, G. TEPHNADZE, *A sharp boundedness result concerning some maximal operators of Vilenkin-Fejér means*, Mediterr. J. Math. **13**, 4 (2016), 1841–1853.

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[15] G. TEPHNADZE, *On the maximal operators of Vilenkin-Fejér means*, Turk. J. Math. **37** (2013), 308–318.
[16] F. SCHIPP, W. R. WADE, P. SIMON, AND J. PÁL, *Walsh Series. An Introduction to Dyadic Harmonic Analysis*, Adam Hilger (Bristol-New York 1990).
[17] P. SIMON, *Investigations with respect to the Vilenkin system*, Annales Univ. Sci. Budapest., Sect. Math. **27** (1985), 87–101.
[18] P. SIMON, *Cesàro summability with respect to two-parameter Walsh system*, Monatsh. Math. **131** (2000), 321–334.
[19] F. WEISZ, *Martingale Hardy spaces and their applications in Fourier analysis*, Springer-Verlang, Berlin, 1994.
[20] F. WEISZ, *Summability of multi-dimensional Fourier series and Hardy space*, Kluwer Academic, Dordrecht, 2002.
[21] F. WEISZ, *Restricted summability of multi-dimensional Vilenkin-Fourier series*, Annales Univ. Sci. Budapest., Sect. Comp. **35** (2011), 305–317.
[22] F. WEISZ, (C,α) *summability of Walsh-Fourier series*, Anal. Math. **27**, 2 (2001), 141–155.
[23] F. WEISZ, *Cesàro summability of two-dimensional Walsh-Fourier series*, Trans. Amer. Math. Soc. **348** (1996), 2169–2181.
[24] F. WEISZ, *Maximal estimates for the* (C,α) *means of* d*-dimensional Walsh-Fourier series*, Proc. Amer. Math. Soc. **128** (1999), 2337–2345.
[25] F. WEISZ, *Summability results of Walsh- and Vilenkin-Fourier series*, Functions, Series, Operators. Alexits Memorial Conference, Budapest, Hungary, 1999, eds. L. Leindler, F. Schipp and J. Szabados (2002), 443–464.
[26] N. YA. VILENKIN, *On a class of complete orthonormal systems*, Amer. Math. Soc. Transl., **28**, 2 (1963), 1–35.
[27] A. ZYGMUND *Trigonometric series*, Cambridge Press, London, 3rd edition, 2002.