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GENERALIZATION OF JENSEN'S AND JENSEN–STEFFENSEN'S INEQUALITIES BY GENERALIZED MAJORIZATION THEOREM

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Abstract. In this paper, we use generalized majorization theorem and give the generalizations of Jensen's and Jensen-Steffensen's inequalities. We present the generalization of converse of Jensen's inequality. We give bounds for the identities related to the generalization of Jensen's inequality by using Čebyšev functionals. We also give Grüss and Ostrowski types inequalities for these functionals. We present mean value theorems and n-exponential convexity which leads to exponential convexity and log-convexity for these functionals. We give some families of functions which enable us to construct a large families of functions that are exponentially convex and also give classes of means.

Mathematics subject classification (2010): 26D15.

Keywords and phrases: Jensen's inequality, Jensen-Steffensen's inequality, majorization, n-convexity, Čebyšev functional, Grüss type inequalities, Ostrowsky-type inequalities, exponentially convex functions, log-convex functions.

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Paper JMI-11-80

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