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SINGULAR INTEGRALS RELATED TO HOMOGENEOUS MAPPINGS IN TRIEBEL–LIZORKIN SPACES

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Abstract. In this note we establish the boundedness for the singular integral operators related to homogeneous mappings with rough kernels in Triebel-Lizorkin spaces. Some previous results are improved and extended substantially. A main ingredient in the proofs is to establish a criterion of boundedness for the convolution type operator in the above function spaces, which presents a systematic treatment for the related singular integral operators.

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