THE JENSEN AND HERMITE–HADAMARD
INEQUALITY ON THE TRIANGLE

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Abstract. We study the functional forms of the most important inequalities concerning convex functions on the triangle. Our intension is to construct the functional form which implies the integral and discrete form of the Jensen inequality, the Fejér, and so the Hermite-Hadamard inequality. To reach this goal, we combine features of positive linear functionals and convex functions.

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