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INTERPOLATION COEFFICIENTS MIXED FINITE ELEMENT METHODS AND L^∞–ERROR ESTIMATES FOR NONLINEAR OPTIMAL CONTROL PROBLEM

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Abstract. In this paper, we investigate L^∞-error estimates for the convex optimal control problem governed by nonlinear elliptic equations using interpolation coefficients mixed finite element methods. By using the interpolation coefficient thought to process the nonlinear term of equations, we present the mixed finite element approximation with interpolated coefficients for nonlinear optimal control problem. We derive L^∞-error estimates for the interpolation coefficients mixed finite element approximation of nonlinear optimal control problem. Finally some numerical examples are given to confirm our theoretical results.

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