Journal of Mathematical Inequalities
Volume 11, Number 4 (2017), 1131–1149
doi:10.7153/jmi-2017-11-84

# MEAN CENTRAL DISTANCE—CENTRAL DISTANCE INEQUALITIES

JIA JIN WEN, SHAN HE WU, JUN YUAN AND TIAN YONG HAN

*Abstract.* By means of the analysis, convex geometry, computer and majorization theories, in the centered 2-surround system $S^{(2)}\{P,\Gamma,l\}$, we establish the following mean central distance–central distance inequalities:

$$\frac{\exp\left(\frac{1}{|\Gamma|}\oint_\Gamma \log \overline{r}_P\right)}{\exp\left(\frac{1}{|\Gamma|}\oint_\Gamma \log r_P\right)} \geqslant \frac{1}{2}\left[\sec\frac{l\pi}{|\Gamma|}+\cot\frac{l\pi}{|\Gamma|}\log\left(\tan\frac{l\pi}{|\Gamma|}+\sec\frac{l\pi}{|\Gamma|}\right)\right]$$

and

$$\frac{\left(\frac{1}{|\Gamma|}\oint_\Gamma \overline{r}_P^2\right)^{1/2}}{\frac{1}{|\Gamma|}\oint_\Gamma r_P} \geqslant \frac{1}{2}\left[\sec\frac{l\pi}{|\Gamma|}+\cot\frac{l\pi}{|\Gamma|}\log\left(\tan\frac{l\pi}{|\Gamma|}+\sec\frac{l\pi}{|\Gamma|}\right)\right] \text{ when } 0<\angle APA_+\leqslant\tau,$$

where $\tau = 2.49342812654089\ldots$, and $\tau/2$ is the unique real root of the following equation:

$$\frac{\mathrm{d}^2[\sec\theta+\cot\theta\log(\tan\theta+\sec\theta)]}{\mathrm{d}\theta^2}=0,\ \theta\in\left(0,\frac{\pi}{2}\right).$$

We also demonstrate the applications of our results, and obtain the $N$–mean central distance – central distance inequality and the mean central distance–central distance–limit inequality.

*Mathematics subject classification* (2010): 26D15, 26E60, 51K05, 52A40.
*Keywords and phrases*: Isoperimetric inequality, $p$-mean, surround system, mean central distance, Schur-convex function.

## REFERENCES

[1] J. J. WEN AND C. B. GAO, *Geometric inequalities involving the central distance of the centered 2-surround system*, Acta. Math. Sinica., **51** (4) (2008), 815–832, (in Chinese).
[2] C. B. GAO AND J. J. WEN, *Theory of surround system and associated inequalities*, Comput. Math. Appl., **63** (12) (2012), 1621–1640.
[3] C. B. GAO AND J. J. WEN, *Theories and inequalities on the satellite system*, ISRN Math. Anal., **2011**, Article ID 909261, 22 pages.
[4] J. J. WEN, J. YUAN AND S. H. WU, *Isoperimetric inequalities in surround system and space science*, J. Inequal. Appl., **2016**, 74 (2016), 28 pages.
[5] J. J. WEN AND W. L. WANG, *The inequalities involving generalized interpolation polynomial*, Comput. Math. Appl., **56** (4) (2008), 1045–1058.
[6] J. J. WEN AND S. S. CHENG, *Closed balls for interpolating quasi-polynomials*, Comput. Appl. Math., **30** (3) (2011), 545–570.
[7] P. S. BULLEN, P. S. MITRINOVIĆ AND P. M. VASIĆ, *Means and their inequalities*, Reidel: Dordrecht/Boston/Lancaster/Toyyo, 1988.
[8] J. J. WEN, J. YUAN AND S. F. YUAN, *An optimal version of an inequality involving the third symmetric means*, Proc. Indian Acad. Sci. Math. Sci., **118** (4) (2008), 505–516.
[9] C. B. GAO AND J. J. WEN, *A dimensionality reduction principle on the optimization of function*, J. Math. Inequal., **7** (3) (2013), 357–375.


Paper JMI-11-84

[10] J. J. WEN AND W. L. WANG, *The optimization for the inequalities of power means*, J. Inequal. Appl., **2006**, Article ID 46782, 25 pages.
[11] J. J. WEN AND W. L. WANG, *Chebyshev type inequalities involving permanents and their applications*, Linear Algebra Appl., **422** (1) (2007), 295–303.
[12] J. J. WEN, T. Y. HAN AND S. S. CHENG, *Inequalities involving Dresher variance mean*, J. Inequal. Appl., **2013**, 366 (2013), 29 pages.
[13] D. KING-HELE, *Theory of satellite orbits in an atmosphere*, Butterworths, London, 1964.
[14] F. DELLA PIETRA AND N. GAVITONE, *The Relative isoperimetric inequality: the anisotropic case*, J. Convex Anal., **20** (1) (2013), 157–180.
[15] F. R. K. CHUNG AND P. TETALI, *Isoperimetric inequalities for Cartesian products of graphs*, Combin. Probab. Comput. **7** (2) (1998), 141–148.
[16] J. J. WEN AND B. J. ZHOU, *The isoperimetric inequality in* $\mathbb{R}^3$, J. Chengdu University (Natural Science), **25** (4) (2006), 241–246, (in Chinese).
[17] A. W. MARSHALL AND I. OLKIN, *Inequalities: Theory of majorization and its applications*, Academic Press, New York, 1979.
[18] S. H. WU, *On the weighted generalization of the Hermite-Hadamard inequality and its applications*, Rocky Mountain J. Math., **39** (5) (2009), 1741–1749.
[19] J. J. WEN, J. E. PEČARIĆ AND Y. H. TIAN, *Weak monotonicity and Chebyshev type inequality*, Math. Inequal. Appl., **18** (1) (2015), 217–231.
[20] J. J. WEN AND Z. H. ZHANG, *Jensen type inequalities involving homogeneous polynomials*, J. Inequal. Appl., **2010**, Article ID 850215, 21 pages.
[21] J. J. WEN, Y. HUANG AND S. S. CHENG, *Theory of* $\phi$ *-Jensen variance and its applications in higher education*, J. Inequal. Appl., **2015**, 270 (2015), 40 pages.
[22] J. J. WEN, C. B. GAO AND W. L. WANG, *Inequalities of J-P-S-F type*, J. Math. Inequal., **7** (2) (2013), 213–225.
[23] J. E. PEČARIĆ, F. PROSCHAN AND Y. L. TONG, *Convex functions*, Academic Press, Boston, MA, 1992.
[24] J. J. WEN, *The inequalities involving Jensen functions*, J. Sys. Sci. Math. Sci., **27** (2) (2007), 208–218, (in Chinese).
[25] D. L. REN, *Topics in integral geometry*, World Scientific Publishing Co., River Edge, NJ, 1994.