

A GENERALIZED GRONWALL–BELLMAN TYPE DELAY INTEGRAL INEQUALITY WITH TWO INDEPENDENT VARIABLES ON TIME SCALES

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Abstract. Using a technique of monotonization, this paper investigates a generalized Gronwall–Bellman type delay integral inequality with two independent variables on time scales. The result not only unifies some continuous inequalities and their discrete analogues but also extends some known integral inequalities on time scales. An application to the estimation of solutions of delay dynamic integral equations on time scales is also given.

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