

UNIQUENESS THEOREMS ON FUNCTIONAL INEQUALITIES CONCERNING CUBIC–QUADRATIC–ADDITIVE EQUATION

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Abstract. We prove uniqueness theorems concerning the functional inequalities in connection with an n -dimensional cubic-quadratic-additive equation $\sum_{i=1}^m c_i f(a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n) = 0$ by applying the direct method.

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