

ANOTHER LOOK AT VORONOVSKAJA TYPE FORMULAS

FADEL NASAIREH AND IOAN RAŞA

Abstract. Voronovskaja type formulas are usually established for sequences of positive linear operators, i.e., operators P_n with $P_n f \geq 0$ whenever the function f is positive. The aim of this paper is twofold. First, we obtain some general formulas concerning compositions of operators on Banach spaces, without any assumption of positivity. Next, we establish Voronovskaja type formulas for operators which are manifestly nonpositive. Combining these two approaches we recover some known results and obtain new applications. The final section is devoted to inequalities accompanying our formulas.

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