

## MATRIX RICHARD INEQUALITY VIA THE GEOMETRIC MEAN

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*Abstract.* In this paper, we show the matrix version of Richard inequality by virtue of Cauchy-Schwartz type inequalities via the matrix geometric mean. As an application, we show a matrix Buzano inequality.

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