

A MORE ACCURATE MULTIDIMENSIONAL HARDY–HILBERT TYPE INEQUALITY WITH A GENERAL HOMOGENEOUS KERNEL

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Abstract. In this paper, by the use of the weight coefficients, the transfer formula, Hermite-Hadamard's inequality and the technique of real analysis, a more accurate multidimensional Hardy-Hilbert-type inequality with a general homogeneous kernel and a best possible constant factor is given, which is an extension of some published results. Moreover, the equivalent forms, the operator expressions and some particular examples are considered.

Mathematics subject classification (2010): 26D15, 47A05.

Keywords and phrases: Hardy-Hilbert-type inequality, weight coefficient, Hermite-Hadamard's inequality, equivalent form, operator.

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