

NONLINEAR RETARDED INTEGRAL INEQUALITIES ON TIME SCALES AND THEIR APPLICATIONS

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Abstract. In this paper, some new nonlinear retarded integral inequalities on time scales are established, which provide a handy tool in the study of some retarded integral equations and dynamic equations on time scales. The results unify and extend some continuous inequalities and their corresponding discrete analogues. Some applications are also presented to illustrate the usefulness of some of our results.

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