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MAXIMAL NUMERICAL RANGE OF A COMPACT SET AND APPLICATIONS TO SOME DRAGOMIR'S INEQUALITIES

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Abstract. Let K, A be respectively a compact and an element of $B(H)$ the algebra of all bounded linear operators acting on a complex Hilbert space H. In this paper we define the maximal numerical range of the set $A^*K = \{A^*B : B \in K\}$ relatively to K by

$$W_K(A^*K) = co(\bigsqcup_{B \in K} W_B(A^*B)).$$

Where $W_B(A^*B)$ is the maximal numerical range of A^*B relatively to B defined by Magajna [6] and which coincides with the maximal numerical range $W_0(B)$ of B defined by Stampfli [7] if A is the unit element I. Our new definition will generalize the results of Stampfli [7] and Barraa-Boumazguour [1] over the distance of an element B to $Vect(A)$. It also will generalize and improve several inequalities established by Dragomir [4, 5] linking the norm and the numerical radius of B.

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