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ESTIMATES FOR THE FIRST EIGENVALUE FOR p–LAPLACIAN WITH MIXED BOUNDARY CONDITIONS

KUI WANG

Abstract. In this article, we consider eigenvalue problems on domains with an interior hole. Precisely, we show a Cheng-type inequality on manifolds, and certain Faber-Krahn inequalities on space forms. Besides, we obtain upper and lower bounds of the eigenvalue through the classic Dirchlet eigenvalue, implying convergence of the eigenvalue as the hole tends to $\emptyset$.

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