ON REVERSES OF THE GOLDEN–THOMPSON TYPE INEQUALITIES

MOHAMMAD BAGHER GHAEMI, VENUS KALEIBARY AND SHIGERU FURUICHI

Abstract. In this paper we present some reverses of the Golden-Thompson type inequalities: Let $H$ and $K$ be Hermitian matrices such that $e^{tH} \preceq e^{sH} e^{K} e^{sH}$ for some scalars $s \leq t$, and $\alpha \in [0, 1]$. Then for all $p > 0$ and $k = 1, 2, \ldots, n$

$$\lambda_k(e^{((1-\alpha)H+\alpha K)}) \leq (\max\{S(e^{e^p}), S(e^{e^p})\})^{\frac{1}{p}} \lambda_k(e^{e^p})^{\frac{1}{p}},$$

where $A^{\frac{1}{2}} B A^{\frac{1}{2}} = A^{\frac{1}{2}} (A^{-\frac{1}{2}} B A^{-\frac{1}{2}})^{\alpha} A^{\frac{1}{2}}$ is $\alpha$-geometric mean, $S(t)$ is the so called Specht ratio and $\preceq_{ols}$ is the so called Olson order. The same inequalities are also provided with other constants. The obtained inequalities improve some known results.


Keywords and phrases: Ando-Hiai inequality, Golden-Thompson inequality, eigenvalue inequality, geometric mean, Olson order, Specht ratio, generalized Kantorovich constant, unitarily invariant norm.

REFERENCES


