

ON REVERSES OF THE GOLDEN–THOMPSON TYPE INEQUALITIES

MOHAMMAD BAGHER GHAEMI, VENUS KALEIBARY AND SHIGERU FURUICHI

Abstract. In this paper we present some reverses of the Golden-Thompson type inequalities: Let H and K be Hermitian matrices such that $e^s e^H \preceq_{ols} e^K \preceq_{ols} e^t e^H$ for some scalars $s \leq t$, and $\alpha \in [0, 1]$. Then for all $p > 0$ and $k = 1, 2, \dots, n$

$$\lambda_k(e^{(1-\alpha)H+\alpha K}) \leq (\max\{S(e^{sp}), S(e^{tp})\})^{\frac{1}{p}} \lambda_k(e^{pH} \#_{\alpha} e^{pK})^{\frac{1}{p}},$$

where $A \#_{\alpha} B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}B^{\frac{1}{2}}A^{-\frac{1}{2}})^{\alpha}A^{\frac{1}{2}}$ is α -geometric mean, $S(t)$ is the so called Specht ratio and \preceq_{ols} is the so called Olson order. The same inequalities are also provided with other constants. The obtained inequalities improve some known results.

Mathematics subject classification (2010): 15A42, 15A60, 47A63.

Keywords and phrases: Ando–Hiai inequality, Golden-Thompson inequality, eigenvalue inequality, geometric mean, Olson order, Specht ratio, generalized Kantorovich constant, unitarily invariant norm.

REFERENCES

- [1] T. ANDO AND F. HIAI, *Log-majorization and complementary Golden-Thompson type inequalities*, Linear Algebra Appl. **197/198** (1994), 113–131.
- [2] R. BHATIA, *Matrix Analysis*, Grad. Texts in Math., vol. **169**, Springer-Verlag, 1997.
- [3] J.-C. BOURIN AND Y. SEO, *Reverse inequality to Golden-Thompson type inequalities: comparison of e^{A+B} and $e^A e^B$* , Linear Algebra Appl. **426** (2007), 312–316.
- [4] S. FURUICHI AND N. MINCULETE, *Alternative reverse inequalities for Young’s inequality*, J. Math. Inequal. **5** (2011), 595–600.
- [5] T. FURUTA, J. MIĆIĆ, J. E. PEČARIĆ AND Y. SEO, *Mond–Pečarić method in operator inequalities*, Monographs in Inequalities 1, Element, Zagreb, 2005.
- [6] M. B. GHAEMI AND V. KALEIBARY, *Some inequalities involving operator monotone functions and operator means*, Math. Inequal. Appl. **19** (2016), 757–764.
- [7] M. B. GHAEMI AND V. KALEIBARY, *Eigenvalue inequalities related to the Ando–Hiai inequality*, Math. Inequal. Appl. **20** (2017), 217–223.
- [8] F. HIAI AND D. PETZ, *The Golden-Thompson trace inequality is complemented*, Linear Algebra Appl. **181** (1993), 153–185.
- [9] F. KUBO AND T. ANDO, *Means of positive linear operators*, Math. Ann. **246** (1980), 205–224.
- [10] A. W. MARSHALL AND I. OLKIN, *Matrix versions of the Cauchy and Kantorovich inequalities*, Aequationes Math. **40** (1990), 89–93.
- [11] M. P. OLSON, *The selfadjoint operators of a von Neumann algebra from a conditionally complete lattice*, Proc. Amer. Math. Soc. **28** (1971), 537–544.
- [12] Y. SEO, *Reverses of the Golden–Thompson type inequalities due to Ando–Hiai–Petz*, Banach J. Math. Anal. **2** (2008), 140–149.
- [13] Y. SEO, *On a reverse of Ando–Hiai inequality*, Banach J. Math. Anal. **4** (2010), 87–91.
- [14] W. SPECHT, *Zur Theorie der elementaren Mittel*, Math. Z. **74** (1960), 91–98.