

## CAUCHY AND PÓLYA–SZEGŐ TYPE INEQUALITIES INVOLVING TWO LINEAR ISOTONIC FUNCTIONALS

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**Abstract.** We consider inequalities which involve two linear isotonic functionals. We give two variants of the Cauchy inequality and few Pólya–Szegő type inequalities in which functions with variable bounds occurred. With help of these inequalities we are able to obtain a new bound for the Chebyshev difference and give some particular cases. Connections of the presented results with earlier results involving fractional integrals are also pointed out.

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