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CAUCHY AND PÓLYA–SZEGÖ TYPE INEQUALITIES INVOLVING TWO LINEAR ISOTONIC FUNCTIONALS

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Abstract. We consider inequalities which involve two linear isotonic functionals. We give two variants of the Cauchy inequality and few Pólya-Szegö type inequalities in which functions with variable bounds occurred. With help of these inequalities we are able to obtain a new bound for the Chebyshev difference and give some particular cases. Connections of the presented results with earlier results involving fractional integrals are also pointed out.

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