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SOME INEQUALITIES RELATED TO p–SCHATTEN NORM

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Abstract. In this paper, we investigate the known operator inequalities for the p-Schatten norm and obtain some refinements of these inequalities when parameters taking values in different regions. Let $A_1,\cdots,A_n,B_1,\cdots,B_n\in B_p(H)$ such that $\Sigma_{i,j=1}^n A_i^*B_j=0$. Then for $0<p\leqslant 2$, $p\geqslant\lambda>0$ and $\mu\geqslant 2$,

$$n^{2(\frac{1}{p}-\frac{1}{\lambda})}\Big(\sum_{i,j=1}^n\|A_i\pm B_j\|_p^{\lambda}\Big)^{\frac{1}{\lambda}}\leqslant n^{\frac{2}{p}-\frac{1}{2}}\Big(\sum_{i=1}^n\|A_i\|_p^p+\sum_{i=1}^n\|B_i\|_p^p\Big)^{\frac{1}{p}}$$
$$\leqslant 2^{\frac{1}{p}-\frac{2}{p\mu}}n^{\frac{3}{p}-\frac{2}{p\mu}-\frac{1}{2}}\Big(\sum_{i=1}^n\|A_i\|_{\frac{p\mu}{2}}^{\frac{p\mu}{2}}+\sum_{i=1}^n\|B_i\|_{\frac{p\mu}{2}}^{\frac{p\mu}{2}}\Big)^{\frac{2}{p\mu}}.$$

For $p\geqslant 2$, $p\leqslant\lambda$ and $0<\mu\leqslant 2$, the inequalities are reversed. Moreover, we get some applications of our results.

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