

## SOME INEQUALITIES RELATED TO $p$ -SCHATTEN NORM

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**Abstract.** In this paper, we investigate the known operator inequalities for the  $p$ -Schatten norm and obtain some refinements of these inequalities when parameters taking values in different regions. Let  $A_1, \dots, A_n, B_1, \dots, B_n \in B_p(H)$  such that  $\sum_{i,j=1}^n A_i^* B_j = 0$ . Then for  $0 < p \leq 2$ ,  $p \geq \lambda > 0$  and  $\mu \geq 2$ ,

$$\begin{aligned} n^{2(\frac{1}{p}-\frac{1}{\lambda})} \left( \sum_{i,j=1}^n \|A_i \pm B_j\|_p^\lambda \right)^{\frac{1}{\lambda}} &\leq n^{\frac{2}{p}-\frac{1}{2}} \left( \sum_{i=1}^n \|A_i\|_p^p + \sum_{i=1}^n \|B_i\|_p^p \right)^{\frac{1}{p}} \\ &\leq 2^{\frac{1}{p}-\frac{2}{p\mu}} n^{\frac{3}{p}-\frac{2}{p\mu}-\frac{1}{2}} \left( \sum_{i=1}^n \|A_i\|_p^{\frac{p\mu}{2}} + \sum_{i=1}^n \|B_i\|_p^{\frac{p\mu}{2}} \right)^{\frac{2}{p\mu}}. \end{aligned}$$

For  $p \geq 2$ ,  $p \leq \lambda$  and  $0 < \mu \leq 2$ , the inequalities are reversed. Moreover, we get some applications of our results.

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