SOME INEQUALITIES RELATED TO $p$–SCHATTEN NORM

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Abstract. In this paper, we investigate the known operator inequalities for the $p$-Schatten norm and obtain some refinements of these inequalities when parameters taking values in different regions. Let $A_1, \ldots, A_n, B_1, \ldots, B_n \in B_p(H)$ such that $\sum_{i,j=1}^n A_i^* B_j = 0$. Then for $0 < p \leq 2$, $p \geq \lambda > 0$ and $\mu \geq 2$,

$$n^{2(\frac{1}{p} - \frac{1}{\lambda})}(\sum_{i,j=1}^n \|A_i \pm B_j\|_p^\lambda)^{\frac{1}{p}} \leq n^{2(\frac{1}{\mu} - \frac{1}{\mu'})}(\sum_{i=1}^n \|A_i\|_p^\mu + \sum_{i=1}^n \|B_i\|_p^\mu)^{\frac{1}{p}}$$

$$\leq 2^{\frac{1}{p} - \frac{1}{\lambda}} n^{\frac{1}{p} - \frac{1}{\mu'}}(\sum_{i=1}^n \|A_i\|_p^{\mu} + \sum_{i=1}^n \|B_i\|_p^{\mu})^{\frac{1}{p'}}.$$

For $p \geq 2$, $p \leq \lambda$ and $0 < \mu \leq 2$, the inequalities are reversed. Moreover, we get some applications of our results.


Keywords and phrases: $p$-Schatten norm, operator inequality, convexity, concavity, orthogonality.

REFERENCES