

BOUNDEDNESS OF MARCINKIEWICZ INTEGRALS ON HARDY SPACES H^p OVER NON-HOMOGENEOUS METRIC MEASURE SPACES

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Abstract. Under the assumption that (\mathcal{X}, d, μ) is a non-homogeneous metric measure space, the authors prove that the Marcinkiewicz integral operator is bounded from the molecular Hardy space $\widetilde{H}_{mb,\rho}^{p,q,\gamma,\varepsilon}(\mu)$ (or the atomic Hardy space $\widetilde{H}_{atb,\rho}^{p,q,\gamma}(\mu)$) into the Lebesgue space $L^p(\mu)$. To this end, some boundedness criteria on these Hardy spaces are established.

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