Journal of Mathematical Inequalities
Volume 12, Number 2 (2018), 365–377
doi:10.7153/jmi-2018-12-27

OPTIMAL BOUNDS FOR THE FIRST SEIFFERT MEAN IN TERMS OF THE CONVEX COMBINATION OF THE LOGARITHMIC AND NEUMAN–SÁNDOR MEAN

JIAN-JUN LEI, JING-JING CHEN AND BO-YONG LONG

Abstract. In this paper, we find the least value α and the greatest value β such that the double inequality

$$\alpha L(a,b)+(1-\alpha)M(a,b)<P(a,b)<\beta L(a,b)+(1-\beta)M(a,b)$$

holds for all $a,b>0$ with $a\neq b$, where $L(a,b),M(a,b)$ and $P(a,b)$ are the logarithmic, the Neuman-Sándor, and the first Seiffert means of two positive numbers a and b, respectively.

Mathematics subject classification (2010): 26E60.
Keywords and phrases: The first Seiffert mean, Neuman-Sándor mean, logarithmic mean.

REFERENCES

[1] H. ALZER AND S. L. QIU, *Inequalities for means in two variables*, Arch. Math. (Basel) **80**, 2 (2003), 201–215.
[2] J. M. BORWEIN AND P. B. BORWEIN, *Inequalities for compound mean iterations with logarithmic asymptotes*, J. Math. Anal. Appl, **177**, 2 (1993), 572–582.
[3] F. BURK, *The geometric, logarithmic, and arithmetic mean inequality*, Amer. Math. Monthly **94**, 6 (1987), 527–528.
[4] Y. M. CHU AND B. Y. LONG, *Bounds of the Neuman-Sándor mean using power and identric means*, Abstr. Appl. Anal. 2013, Article ID 832591, 6 pages.
[5] Y.-M. CHU, M.-Y. SHI AND Y.-P. JIANG, *Optimal inequalities for the power, harmonic and logarithmic means*, Bull. Math. **38**, 3(2012), 597–606.
[6] Y. M. CHU, M. K. WANG AND G. D. WANG, *The optimal generalized logarithmic mean bounds for Seiffert's mean*, Acta. Math. Sci, **32** , 4(2012), 1619–1626.
[7] P. A. HÄSTÖ, *Optimal inequalities between Seiffert's mean and power means*, Mathematical Inequalities & Applications, **7**, 1(2004), 47–53.
[8] A. A. JAGERS, *Solution of problem 887*, Nieuw Archief voor Wiskunde, **12**, 4(1994), 230–231.
[9] W.-D. JIANG AND F. QI, *Some sharp inequalities involving Seiffert and other means and their concise proofs*, Mathematical Inequalities & Applications, **15**, 4(2012), 1007–1017.
[10] Y. M. LI, B. Y. LONG AND Y. M. CHU, *Sharp bounds for the Neuman-Sándor mean in terms of generalized logarithmic mean*, J. Math. Inequal. **6**, 4(2012), 567–577.
[11] E. NEUMAN, *A note on a certain bivariate mean*, J. Math. Inequal. **6**, 4(2012), 637–643.
[12] E. NEUMAN AND J. SÁNDOR, *On the Schwab-Borchardt mean*, Math. Pannon. **14**, 2(2003), 253–266.
[13] E. NEUMAN AND J. SÁNDOR, *On the Schwab-Borchardt mean*, Math II, Pannon. **17**, 1(2006), 49–59.
[14] W. M. QIAN AND Z. H. SHEN, *Inequalities between power means and convex combinations of the harmonic and logarithmic means*, Journal of Applied Mathematics, **2012** (2012), Article ID 471096, 14 pages.
[15] J. SÁNDOR, *On certain inequalities for means III*, Arch. Math, **76**, 1 (2001), 30–40.
[16] M. Y. SHI, Y. M. CHU AND Y. P. JIANG, *Optimal inequalities related to the power, harmonic and identric means*, Acta Math Sci, **31A**, 5 (2011), 1377–1384.
[17] A. WITKOWSKI, *Optimal weighted harmonic interpolations between Seiffert means*, Colloquium Mathematicum, **130**, 2(2013), 265–279.

Paper JMI-12-27

[18] N. G. ZHENG, X. M. ZHANG AND Y. M. CHU, *Convexity and geometrical convexity of exponential and logarithmic means in N variables*, Acta. Math. Sci, **28A**, 6(2008), 1173–1180.