

A NEW FORM OF HILBERT INTEGRAL INEQUALITY

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Abstract. In this paper by estimating the triple integral $\int_0^\infty \int_0^\infty \int_0^\infty \frac{f(x,y)g(z)}{(x+y+z)^\lambda} dx dy dz$, we introduce a new form of the Hilbert inequality for three variables with a best constant factor. The reverse form and some equivalent forms are also considered.

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REFERENCES

- [1] L. E. AZAR, *Some Extensions of Hilbert's integral inequality*, J. Math. Inequal. **5**, 1 (2011), 131–140.
- [2] I. BRNETIĆ, M. KRNIĆ AND J. PEČARIĆ, *Multiple Hilbert and Hardy-Hilbert inequalities with non-conjugate parameters*, Bull. Aust. Math. Soc. **71**, 3 (2005), 447–457.
- [3] M. T. GARAYEV, M. GÜRDAL AND A. OKUDAN, *Hardy-Hilbert's inequality and power inequalities for Berezin numbers of operators*, Math. Inequal. Appl. **19**, 3 (2016), 883–891.
- [4] G. H. HARDY, J. E. LITTLEWOOD AND G. POLYA, *Inequalities*, Cambridge Univ. Press, London, 1952.
- [5] M. KRNIĆ, *A refined discrete Hilbert inequality via the Hermite-Hadamard inequality*, Comput. Math. Appl. **63**, 12 (2012), 1587–1596.
- [6] M. KRNIĆ, G. MINGZHE, J. PEČARIĆ AND G. XUEMEI, *On the best constant in Hilbert's inequality*, Math. Ineq. and Appl. **8**, 2 (2005), 317–329.
- [7] M. KRNIĆ AND J. PEČARIĆ, *Extension of Hilbert's inequality*, J. Math. Anal. Appl. **324**, 1 (2006), 150–160.
- [8] M. KRNIĆ, J. PEČARIĆ, I. PERIĆ AND P. VUKOVIĆ, *Recent Advances in Hilbert-type Inequalities*, Element, Zagreb, 2012.
- [9] M. KRNIĆ, J. PEČARIĆ AND P. VUKOVIĆ, *On some higher-dimensional Hilbert's and Hardy-Hilbert's integral inequalities with parameters*, Math. Inequal. Appl. **11**, 4 (2008), 701–716.
- [10] G. MINGZHE AND B. YANG, *On the extended Hilbert's inequality*, Proc. Amer. Math. Soc. **126**, 3 (1998), 751–759.
- [11] D. S. MITRINOVIĆ, J. PEČARIĆ AND A. M. FINK, *Classical and new inequalities in analysis*, Kluwer Academic, Publishers, Dordrecht/Boston/London, 1993.
- [12] B. YANG, *On a new multiple extension of Hilbert's integral inequality*, J. Inequal. Pure Appl. Math. **6**, 2 (2005), 1–8.
- [13] B. YANG, *The norm of operator and Hilbert-type inequalities*, Science press, Beijing, 2009.
- [14] B. YANG AND T. M. RASSIAS, *On the way of weight coefficients and research for the Hilbert-type inequalities*, Math. Inequal. Appl. **6**, 4 (2003), 625–658.