

## GRADIENT ESTIMATES FOR A NONLINEAR ELLIPTIC EQUATION ON COMPLETE NONCOMPACT RIEMANNIAN MANIFOLD

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*Abstract.* Let  $(M, g)$  be an  $n$ -dimensional complete noncompact Riemannian manifold (with possibly empty boundary). We derive local and global gradient estimates on positive solutions  $u(x)$  to the following nonlinear elliptic equation

$$\Delta u(x) + au^s(x) + \lambda(x)u(x) = 0, \quad x \in M,$$

where  $a$  and  $s$  are constants,  $a \in \mathbb{R} \setminus \{0\}$ ,  $s > 1$  and  $\lambda(x)$  is bounded on  $M$ . Our gradient estimates yield differential Harnack inequalities as an application. This paper extends results of Y. Yang [17] and J. Li [11, Theorem 3.1].

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