

PARTIAL SUMS OF MITTAG–LEFFLER FUNCTION

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Abstract. In the present investigation, Mittag-Leffler function with their normalization are considered. In this paper, we will study the ratio of a function of the form (??) to its sequence of partial sums $(\mathbb{E}_{\lambda, \mu})_n(z) = z + \sum_{k=1}^n \frac{\Gamma(\mu)}{\Gamma(\lambda k + \mu)} z^{k+1}$. We will determine lower bounds for $\Re \left\{ \frac{\mathbb{E}_{\lambda, \mu}(z)}{(\mathbb{E}_{\lambda, \mu})_n(z)} \right\}$, $\Re \left\{ \frac{(\mathbb{E}_{\lambda, \mu})_n(z)}{\mathbb{E}_{\lambda, \mu}(z)} \right\}$, $\Re \left\{ \frac{\mathbb{E}'_{\lambda, \mu}(z)}{(\mathbb{E}'_{\lambda, \mu})_n(z)} \right\}$ and $\Re \left\{ \frac{(\mathbb{E}'_{\lambda, \mu})_n(z)}{\mathbb{E}'_{\lambda, \mu}(z)} \right\}$. Results obtained are new and their usefulness are depicted by deducing several interesting examples.

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