

GENERAL THEOREMS ON EXPONENTIAL AND ROSENTHAL'S INEQUALITIES AND ON COMPLETE CONVERGENCE

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Abstract. Exponential inequalities are obtained under general conditions. Then it is shown that an exponential inequality implies both Rosenthal's inequality and complete convergence of sums of random variables. The general results are applied to weakly orthant dependent sequences.

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