SOME GENERALIZATIONS OF NUMERICAL RADIUS ON
OFF–DIAGONAL PART OF 2 × 2 OPERATOR MATRICES

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Abstract. We generalize several inequalities involving powers of the numerical radius for off-
diagonal part of 2 × 2 operator matrices of the form

\[ T = \begin{bmatrix} 0 & B \\ C & 0 \end{bmatrix}, \]

where \( B, C \) are two operators.

In particular, if \( T = \begin{bmatrix} 0 & B \\ C & 0 \end{bmatrix} \), then we get

\[
\frac{1}{2^r(r-1)} \max \{ \| \mu \|, \| \eta \| \} \leq w^r(T) \leq \frac{1}{2^r+1} \max \{ \| \mu \|, \| \eta \| \},
\]

where \( r \geq 2 \), \( \mu = |(C - B^*) + i(C + B^*)| + |(B^* - C) + i(C + B^*)| \) and \( \eta = |(B - C^*) + i(B + C^*)| + |(C^* - B) + i(B + C^*)| \).


Keywords and phrases: Cartesian decomposition, Jensen inequality, numerical radius, off-diagonal
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