

SOME GENERALIZATIONS OF NUMERICAL RADIUS ON OFF-DIAGONAL PART OF 2×2 OPERATOR MATRICES

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Abstract. We generalize several inequalities involving powers of the numerical radius for off-diagonal part of 2×2 operator matrices of the form $T = \begin{bmatrix} 0 & B \\ C & 0 \end{bmatrix}$, where B, C are two operators.

In particular, if $T = \begin{bmatrix} 0 & B \\ C & 0 \end{bmatrix}$, then we get

$$\frac{1}{2^{\frac{1}{2}(r-1)}} \max\{\|\mu\|, \|\eta\|\} \leq w^r(T) \leq \frac{1}{2^{r+1}} \max\{\|\mu\|, \|\eta\|\},$$

where $r \geq 2$, $\mu = |(C - B^*) + i(C + B^*)|^r + |(B^* - C) + i(C + B^*)|^r$ and $\eta = |(B - C^*) + i(B + C^*)|^r + |(C^* - B) + i(B + C^*)|^r$.

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