

A PROOF OF AN OPEN PROBLEM OF YUSUKE NISHIZAWA FOR A POWER–EXPONENTIAL FUNCTION

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Abstract. This paper presents a proof of the following conjecture, stated by Yusuke Nishizawa in [Appl. Math. Comput. 269, (2015), 146–154.]: for $0 < x < \pi/2$ the inequality $\frac{\sin x}{x} > \left(\frac{2}{\pi} + \frac{\pi-2}{\pi^3}(\pi^2 - 4x^2)\right)^{\theta(x)}$ holds, where $\theta(x) = -\frac{(\pi^3 - 24\pi + 48)x^3}{3(\pi-2)\pi^3} + \frac{\pi^3}{24(\pi-2)}$.

Mathematics subject classification (2010): 26D05, 41A10.

Keywords and phrases: Mixed logarithmic-trigonometric polynomial functions and inequalities.

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