Journal of Mathematical Inequalities
Volume 12, Number 2 (2018), 487–506
doi:10.7153/jmi-2018-12-36

THE DUAL ORLICZ–BRUNN–MINKOWSKI INEQUALITY FOR CONCAVE FUNCTIONS

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Abstract. In this paper, we define the Orlicz radial sum and dual Orlicz mixed quermassintegral for concave functions, and then establish the dual Orlicz-Brunn-Minkowski inequality for concave functions.

Mathematics subject classification (2010): 52A40, 52A20.
Keywords and phrases: Star body, Orlicz radial sum, dual Orlicz mixed volume.

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