

ON APPROXIMATION PROPERTIES OF SOME CLASS POSITIVE LINEAR OPERATORS IN q -ANALYSIS

ERSIN ŞİMŞEK AND TUNCAY TUNÇ

Abstract. This paper is concerned with some sequences of the positive linear operators based on q -Calculus. The approximation properties and the rate of convergence of these sequences of q -discrete type is established by means of the modulus of continuity. Moreover we give Voronovskaya-type theorems. Finally we present some applications such as q -Bernstein operators and q -Meyer-König and Zeller operators.

Mathematics subject classification (2010): 05A30, 41A25, 41A36, 47B38.

Keywords and phrases: q -calculus, generating functions, positive linear operators, rate of convergence.

REFERENCES

- [1] A. ARAL, O. DOĞRU, *Bleimann Butzer and Hahn operators based on q -integers*, J. Inequal. Appl. **2007**, (2007), 12 pp. [Article ID 79410].
- [2] A. ALTIN, O. DOGRU, F. TASDELEN, *The generalization of Meyer-König and Zeller operators by generating functions*, J. Math. Anal. Appl. **312** (2005), 181–194.
- [3] A. ARAL, V. GUPTA, R. P. AGARWAL, *Applications of q -Calculus in Operator Theory*, Springer, New York, 2013.
- [4] A. ARAL, V. GUPTA, *On q -Baskakov type operators*, Demons. Math. **42**, 1 (2009), 109–122.
- [5] A. II'INSKI, S. OSTROVSKA, *Convergence of generalized Bernstein polynomials*, J. Approx. Theor. **116**, (2002), 100–112.
- [6] A. LUPAŞ, *A q -analogue of the Bernstein operator*, University of Cluj-Napoca Seminar on numerical and statistical calculus **9**, (1987), 85–92.
- [7] C. RADU, *On statistical approximation of a general class of positive linear operators extended in q -calculus*, Appl. Math. Comput. **215**, (2009), 2317–2325.
- [8] E. SIMSEK, T. TUNC, *On the Construction of q -Analogues for some Positive Linear Operators*, Filomat, **31**:13 (2017), 4287–4295.
- [9] F. H. JACKSON, *On q -functions and a certain difference operator*, Trans. Roy. Soc. Edin. **46**, (1908), 253–281.
- [10] F. SCHURER, *Positive linear operators in approximation theory*, Mathematical Institute of the Technological University Delft, Report, 1962.
- [11] G. M. PHILLIPS, *Bernstein polynomials based on the q -integers*, Ann. Numer. Math. **4**, (1997), 511–518.
- [12] G. NOWAK, V. GUPTA, *The Rate of Pointwise Approximation of Positive Linear Operators Based on q -Integer*, Ukrainian Math. J. **3**, (2011), 350–360.
- [13] L. CARLITZ, *q -Bernoulli numbers and polynomials*, Duke Math. J. **15**, 4 (1948), 987–1000.
- [14] N. I. MAHMUDOV, P. SABANCIGIL, *q -Parametric Bleimann Butzer and Hahn operators*, J. Inequal. Appl. **2008**, (2008), 15 pp. [Article ID 816367].
- [15] O. AGRATINI, G. NOWAK, *On a generalization of Bleimann, Butzer and Hahn operators based on q -integers*, Math. Comput. Model. **53**, 5–6 (2011), 699–706.
- [16] O. DOĞRU, O. DUMAN, *Statistical approximation of Meyer-König and Zeller operators based on the q -integers*, Publ. Math. Debrecen **68**, (2006), 190–214.

- [17] P. M. RAJKOVIC, M. S. STANKOVIC, S. D. MARINKOVIC, *Mean value theorems in q -calculus*, Math. Vesnic. **54**, (2002), 171–178.
- [18] T. ERNST, *A new notation for q -calculus and a new q -Taylor formula*, Uppsala University Report Depert. Math., (1999), 1–28.
- [19] T. TRIF, *Meyer, König and Zeller operators based on the q -integers*, Rev. Anal. Numer. Theor. Approx. **29**, (2002), 221–229.
- [20] V. A. BASKAKOV, *An example of sequence of linear positive operators in the space of continuous functions*, Dokl. Akad. Nauk. SSSR **113** (1957), 249–251.
- [21] V. KAC, P. CHEUNG, *Quantum Calculus*, Universitext Springer, New York, 2002.
- [22] V. S. VIDENSKII, *On some class of q -parametric positive operators*, Operator Theory: Advances and Applications **158**, (2005), 213–222.
- [23] W. HEPING, *Properties of convergence for the q -Meyer-König and Zeller operators*, J. Math. Anal. Appl. **335**, 2 (2007), 1360–1373.
- [24] W. HEPING, F. MENG, *The rate of convergence of q -Bernstein polynomials for $0 < q < 1$* , J. Approx. Theor. **136**, (2005), 151–158.
- [25] Z. FINTA, V. GUPTA, *Approximation properties of q -Baskakov operators*, Cent. Eur. J. Math. **8**, 1 (2009), 199–211.
- [26] N. L. BRAHA, V. LOKU, H. M. SRIVASTAVA, *Λ^2 -Weighted statistical convergence and Korovkin and Voronovskaya type theorems*, Appl. Math. Comput. **266**, (2015), 675–686.
- [27] U. KADAK, N. L. BRAHA, H. M. SRIVASTAVA, *Statistical weighted \mathcal{B} -summability and its applications to approximation theorems*, Appl. Math. Comput. **302**, (2017), 80–96.
- [28] H. M. SRIVASTAVA, M. MURSALEEN, A. ALOTAIBI, M. NASIRUZZAMAN, A. A. H. AL-ABIED, *Some approximation results involving the q -Szász-Mirakjan-Kantorovich type operators via Dunkl's generalization*, Math. Meth. Appl. Sci. **40** doi : 10.1002/mma.4397, (2017), 5437–5452.
- [29] M. MURSALEEN, A. KHAN, H. M. SRIVASTAVA, K. S. NISAR, *Operators constructed by means of q -Lagrange polynomials and A -statistical approximation*, Appl. Math. Comput. **219**, 12 (2017), 6911–6918.
- [30] M. MURSALEEN, K. J. ANSARI, A. KHAN, *Approximation by Kantorovich Type q -Bernstein-Stancu Operators*, Complex Analysis and Operator Theory. **11**, 1 (2017), 85–107.
- [31] L. NAIM BRAHA, H. M. SRIVASTAVA, S. A. MOHIUDDINE, *A Korovkin's type approximation theorem for periodic functions via the statistical summability of the generalized de la Vallée Poussin mean*, Appl. Math. Comput. **228**, (2014), 162–169.
- [32] V. GUPTA, *Some approximation properties of q -Durrmeyer operators*, Appl. Math. Comput. **197**, 1 (2008), 172–178.
- [33] V. GUPTA, Z. FINTA, *On certain q -Durrmeyer type operators*, Appl. Math. Comput. **209**, (2009), 415–420.
- [34] Ö. DAMLAOĞLU, *Approximation by Kantorovich type q -Bernstein operators*, in Proceedings of the 12th WSEAS International Conference on Applied Mathematics, Cairo, Egypt, (2007), 113–117.
- [35] V. GUPTA, C. RADU, *Statistical approximation properties of q -Baskakov Kantorovich operators*, Cent. Eur. J. Math. **7** (4), (2009), 809–818.