

## OZAKI'S INEQUALITY AND UMEZAWA'S CONDITION FOR MULTIVALENT FUNCTIONS

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*Abstract.* Let  $f(z)$  be analytic in  $|z| < R$ , continuous on  $|z| = R$  and  $f'(z) \neq 0$  on  $|z| = R$ . Then holds Ozaki's inequality that the total variation of  $\arg\{f(z)\}$  on  $|z| = R$  is not more than the total variation of  $\arg\{df(z)\}$  on  $|z| = R$ . Here we consider also Umezawa's condition that

$$-\frac{\alpha}{2\alpha-3} < 1 + \Re e \frac{zf''(z)}{f'(z)} < \alpha \quad |z| < 1$$

follows the univalence of  $f(z)$  in  $|z| < 1$ . In this paper we extended these results for multivalent functions.

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