A NEW CHARACTERIZATION OF CONVEXITY WITH RESPECT TO CHEBYSHEV SYSTEMS

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Abstract. The notion of $n$th order convexity in the sense of Hopf and Popoviciu is defined via the nonnegativity of the $(n + 1)$st order divided differences of a given real-valued function. In view of the well-known recursive formula for divided differences, the nonnegativity of $(n + 1)$st order divided differences is equivalent to the $(n - k - 1)$st order convexity of the $k$th order divided differences which provides a characterization of $n$th order convexity.

The aim of this paper is to apply the notion of higher-order divided differences in the context of convexity with respect to Chebyshev systems introduced by Karlin in 1968. Using a determinant identity of Sylvester, we then establish a formula for the generalized divided differences which enables us to obtain a new characterization of convexity with respect to Chebyshev systems. Our result generalizes that of Wąsowicz which was obtained in 2006. As an application, we derive a necessary condition for functions which can be written as the difference of two functions convex with respect to a given Chebyshev system.


Keywords and phrases: Chebyshev system, generalized convexity, generalized divided difference.

REFERENCES