Journal of Mathematical Inequalities
Volume 12, Number 3 (2018), 605–617
doi:10.7153/jmi-2018-12-46

A NEW CHARACTERIZATION OF CONVEXITY WITH RESPECT TO CHEBYSHEV SYSTEMS

ZSOLT PÁLES AND ÉVA SZÉKELYNÉ RADÁCSI

Abstract. The notion of nth order convexity in the sense of Hopf and Popoviciu is defined via the nonnegativity of the $(n+1)$st order divided differences of a given real-valued function. In view of the well-known recursive formula for divided differences, the nonnegativity of $(n+1)$st order divided differences is equivalent to the $(n-k-1)$st order convexity of the kth order divided differences which provides a characterization of nth order convexity.

The aim of this paper is to apply the notion of higher-order divided differences in the context of convexity with respect to Chebyshev systems introduced by Karlin in 1968. Using a determinant identity of Sylvester, we then establish a formula for the generalized divided differences which enables us to obtain a new characterization of convexity with respect to Chebyshev systems. Our result generalizes that of Wąsowicz which was obtained in 2006. As an application, we derive a necessary condition for functions which can be written as the difference of two functions convex with respect to a given Chebyshev system.

Mathematics subject classification (2010): Primary 26B25, Secondary 39B62.
Keywords and phrases: Chebyshev system, generalized convexity, generalized divided difference.

REFERENCES

[1] A. G. AKRITAS, E. K. AKRITAS, AND G. I. MALASCHONOK, *Various proofs of Sylvester's (determinant) identity*, Math. Comput. Simulation **42** (1996), no. 4–6, 585–593, Symbolic computation, new trends and developments (Lille, 1993) MR1430843.

[2] M. BESSENYEI AND ZS. PÁLES, *On generalized higher-order convexity and Hermite-Hadamard-type inequalities*, Acta Sci. Math. (Szeged) **70** (2004), no. 1-2, 13–24, MR2005e:26012.

[3] F. R. GANTMACHER, *The theory of matrices*, vols. 1, 2, translated by K. A. Hirsch, Chelsea Publishing Co., New York, 1959, MR0107649.

[4] E. HOPF, *Über die Zusammenhänge zwischen gewissen höheren Differenzenquotienten reeller Funktionen einer reellen Variablen und deren Differenzier bar keit seigenschaften*, Ph. D. thesis, Friedrich–Wilhelms–Universität Berlin, 1926.

[5] S. KARLIN, *Total positivity*, vol. I, Stanford University Press, Stanford, California, 1968, MR37#5667.

[6] S. KARLIN AND W. J. STUDDEN, *Tchebycheff systems: With applications in analysis and statistics*, Pure and Applied Mathematics, vol. XV, Interscience Publishers John Wiley & Sons, New York-London-Sydney, 1966, MR34#4757.

[7] T. POPOVICIU, *Les fonctions convexes*, Hermann et Cie, Paris, 1944, MR8,319a.

[8] ZS. PÁLES AND É. SZ. RADÁCSI, *Characterizations of higher-order convexity properties with respect to Chebyshev systems*, Aequationes Math. **90** (2016), no. 1, 193–210, MR3471295.

[9] SZ. WĄSOWICZ, *Some properties of generalized higher-order convexity*, Publ. Math. Debrecen **68** (2006), no. 1–2, 171–182, MR2213549 (2006m:26016).