Journal of Mathematical Inequalities
Volume 12, Number 3 (2018), 619–633
doi:10.7153/jmi-2018-12-47

A NEW GENERALIZATION OF BOAS THEOREM FOR SOME LORENTZ SPACES $\Lambda_q(\omega)$

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Abstract. Let $\Lambda_q(\omega)$, $q>0$, denote the Lorentz space equipped with the (quasi) norm

$$\|f\|_{\Lambda_q(\omega)} := \left(\int_0^1 (f^*(t)\omega(t))^q \frac{dt}{t}\right)^{\frac{1}{q}}$$

for a function f on $[0,1]$ and with ω positive and equipped with some additional growth properties. A generalization of Boas theorem in the form of a two-sided inequality is obtained in the case of both general regular system $\Phi=\{\varphi_k\}_{k=1}^{\infty}$ and generalized Lorentz $\Lambda_q(\omega)$ spaces.

Mathematics subject classification (2010): 46E30, 42A16.
Keywords and phrases: Inequalities, two-sided inequalities, Fourier series, Boas theorem, generalized Lorentz spaces, regular systems, generalized monotone function.

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Paper JMI-12-47

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