

CIRCULAR REARRANGEMENT INEQUALITY

HAIYANG YU

Abstract. This paper presents an analogue of the rearrangement inequality, namely the circular rearrangement inequality. It holds for any finite sequence of real numbers. A volume-invariant packing problem and a combinatorial isoperimetric problem are addressed, as the geometric interpretation of the inequality.

Mathematics subject classification (2010): 26D15, 05D99, 52C99, 90C27.

Keywords and phrases: Rearrangement inequality, circular permutation, geometric interpretation.

REFERENCES

- [1] J. C. BOURIN, *Reverse rearrangement inequality via matrix technics*, J. Inequal. Pure and Appl. Math. **7**, 2 (2006), Article 43.
- [2] A. BURCHARD, *A short course on rearrangement inequalities*, Lecture notes, <http://www.math.toronto.edu/almut/rearrange.pdf>, 2009.
- [3] J. W. S. CASSELS, *An Introduction to the Geometry of Numbers*, Springer-Verlag, Classics in Mathematics, Berlin, 1997.
- [4] W. CHAN, *Stochastic rearrangement inequalities*, J. Multivar. Anal. **23**, 2 (1987), 257–275.
- [5] P. W. DAY, *Rearrangement inequalities*, Canad. J. Math. **24** (1972), 930–943.
- [6] C. DRAGHICI, *Rearrangement inequalities with application to ratios of heat kernels*, Potential Anal. **22**, 4 (2005), 351–374.
- [7] H. HAJAIEI, *Rearrangement inequalities in the discrete setting and some applications*, Nonlinear Anal. Theory, Meth. and Appl. **59**, 3-4 (2010), 1140–1148.
- [8] F. HAMEL, N. NADIRASHVILI, AND E. RUSS, *Rearrangement inequalities and applications to isoperimetric problems for eigenvalues*, Ann. Math. **174**, 2 (2011), 647–755.
- [9] G. H. HARDY, J. E. LITTLEWOOD AND G. PÓLYA, *Inequalities*, Cambridge University Press, International Edition, Cambridge, 1952.
- [10] F. HOLLAND, *Rearrangement Inequalities*, Lecture notes, <http://euclid.ucc.ie/pages/MATHENR/MathEnrichment/rearraniq08.pdf>, 2008.
- [11] A. W. MARSHALL, I. OLKIN, AND B. C. ARNOLD, *Inequalities: Theory of Majorization and Its Applications*, Springer, Springer Series in Statistics, New York, 2011.
- [12] L. MIRSKY, *On the trace of matrix products*, Math. Nachr. **20**, 3-6 (1959), 171–174.
- [13] M. NEWMAN, *Kantorovich's inequality*, J. Res. National Bur. Standards **64B** (1960), 33–34.
- [14] H. RICHTER, *Zur abschätzung von matrizennormen*, Math. Nachr. **18** (1958), 178–187.
- [15] F. RIESZ, *Sur une inégalité intégrale*, J. London Math. Soc. **5** (1930), 162–168.
- [16] L. TIE, K. Y. CAI, AND Y. LIN, *Rearrangement inequalities for Hermitian matrices*, Linear Alg and its Appl **434**, 2 (2011), 443–456.
- [17] J. VON NEUMANN, *Some matrix-inequalities and metrization of matrix-space*, Tomsk. Univ. Rev. **1** (1937), 286–300.