

## INEQUALITIES OF THE JENSEN AND EDMUNDSON-LAH-RIBARIČ TYPE FOR 3-CONVEX FUNCTIONS WITH APPLICATIONS

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**Abstract.** In this paper we derive some Jensen and Edmundson-Lah-Ribarič type inequalities for positive linear functionals and 3-convex functions. Obtained results are then applied to generalized means and power means, as well as to the generalized  $f$ -divergence functional. Examples with Zipf-Mandelbrot law are given.

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