

## LYAPUNOV–TYPE INEQUALITIES FOR NONLINEAR DIFFERENTIAL EQUATION WITH HILFER FRACTIONAL DERIVATIVE OPERATOR

YOUYU WANG AND QICHAO WANG

**Abstract.** In this work, we establish Lyapunov-type inequalities for the fractional boundary value problems with Hilfer fractional derivative operator, the results of this paper are new and generalize and improve some early results in the literature.

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