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# MULTI–DIMENSIONAL HARDY TYPE INEQUALITIES IN HÖLDER SPACES

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*Abstract.* Most Hardy type inequalities concern boundedness of the Hardy type operators in Lebesgue spaces. In this paper we prove some new multi-dimensional Hardy type inequalities in Hölder spaces.

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