

GENERALIZATION OF MAJORIZATION THEOREM-II

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Abstract. This paper begins with a rigorous study of convex functions with the goal of developing the majorization theorems in the form of Taylor representation. In this paper, some new types of Green functions, introduced by Pečarić-Agarwal-Butt-Mehmood (2017) [11] and Taylor's formula, are used to obtain the identities related to majorization type inequalities. We present the monotonicity of the linear functionals deduced from our generalized results by using the family of $(n + 1)$ -convex functions at a point. We give upper bounds and mean value theorems for obtained generalized identities. At the end, we explore some applications.

Mathematics subject classification (2010): 26A51, 26D15, 26D20, 26D99.

Keywords and phrases: Majorization inequality, Taylor's formula, new Green functions, linear functionals, $(n + 1)$ -convex functions at a point, Grüss and Ostrowski-type upper bounds, mean value theorems, n -exponential convexity, applications.

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